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CRITICAL ANALYSIS ON BIFURCATION OF SPR_SODE MODEL FOR THE SPREAD OF DENGUE

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ABSTRACT

A ordinary differential equation with stochastic parameters, called SPR_SODE model for the spread of dengue fever is considered. Critical Values of bifurcation and the boundary of the above said model are discussed. In this paper, the different parameters are considered for further analysis. The bifurcation at the characteristic value of the non-linear eigen value equation is supercritical if $\Gamma_1 > 0$ and subcritical if $\Gamma_1 < 0$. The equilibrium solution pair in the positive octant of R^7 is also discussed.

KEYWORDS: Boundary, Critical Values, ODE, SPR_SODE Model, Stability

INTRODUCTION

Dengue is a disease comes under infectious diseases which is in worldwide. The work of a carrier (i.e) the medium for transmitting is performed by the mosquito, "Aedes Ageypti [7]. There are so many models for such infectious diseases. We need a separate model for such a special disease like dengue fever for better results. The asymptotically stable equilibrium points or equilibrium solutions can be defined as the equilibrium solutions in which solutions that start "near" them move toward the equilibrium solution [1]. In this work, the SPR_SODE model [2], [3], [4] (SPR_Stochastic Ordinary differential Equation model) is considered to analyze further. All the notions of SPR_SODE model [2], [3],[4] are taken for further analysis without any change and the same model is given below.

SPR_SODE MODEL

$$\frac{d}{dt}\left[SS_h\right]=\wp_h+b_h\left[\left[TP\right]_h\right]+L_h\left[\left[RC\right]_h\right]-\tau_h(t)\left[SS\right]_h-\Omega_h\left[\left[TP\right]_h\right]\left[SS\right]_h$$

$$\frac{d}{dt}\left[EX_h\right]=\tau_h(t)\left[SS\right]_h-\xi_h\left[EX\right]_h-\Omega_h\left(\left[TP\right]_h\right)\left[EX\right]_h$$

$$\frac{d}{dt}\left[IF_h\right]=\xi_h\left[EX\right]_h-\left[RC\right]_h\left[IF\right]_h-\Omega_h\left(\left[TP\right]_h\right)\left[RC\right]_h$$

$$\frac{d}{dt}\left[RC_h\right]=\theta_h\left[IF\right]_h-L_h\left[RC\right]_h-\Omega_h\left(\left[TP\right]_h\right)\left[IF\right]_h-\eta\left[IF\right]_h$$

$$\frac{d}{dt}\left[SS_m\right]=[BIR]_m\left[TP\right]_m-\tau_m(t)\left[SS\right]_m-\Omega_m\left(\left[TP\right]_m\right)\left[SS\right]_m$$

$$\frac{d}{dt}\left[EX_m\right]=\tau_m(t)\left[SS\right]_m-\xi_m\left[EX\right]_m-\Omega_m\left(\left[TP\right]_m\right)\left[EX\right]_m$$

$$\frac{d}{dt}\left[IF_m\right]=\xi_m\left[EX\right]_m-\Omega_m\left(\left[TP\right]_m\right)\left[IF\right]_m \tag{A}$$

By converting (A) to fractional quantities and denoting each scaled population by small letters, one can get, [2], [3], [4]

$$\frac{d}{dt}[ex]_h=\frac{\phi_h\phi_m P_{mh}[if]_m}{\phi_m\left[TP\right]_m+\phi_h\left[TP\right]_h}.\left[TP\right]_m.\left[1-[ex]_h-[if]_h-[re]_h\right]-\left[\xi_h+[BIR]_h+\frac{\wp_h}{\left[TP\right]_h}\right][ex]_h+\eta_h[if]_h[ex]_h$$

$$\frac{d}{dt}\left[if\right]_h=\xi_h\left[ex\right]_h-\left(\theta_h+\left[BIR\right]_h+\frac{\wp_h}{\left[TP\right]_h}\right)\left[if\right]_h+\eta_h\left[if\right]_h^2$$

$$\frac{d}{dt}\left[rc\right]_h=\theta_h\left[if\right]_h-\left(L_h+[BIR]_h\frac{\wp_h}{\left[TP\right]_h}\right)\left[rc\right]_h+\eta_h\left[if\right]_h\left[TP\right]_h$$

$$\frac{d}{dt}\left[TP\right]_h=\wp_h+\theta_h\left[TP\right]_h-\left([DID]_h+[DDD]_h\left[TP\right]_h\right).\left[TP\right]_h-\eta_h[if]_h[TP]_h$$

$$\frac{d}{dt}\left[ex\right]_m=\frac{\phi_h\phi_m}{\phi_m\left[TP\right]_m+\phi_h\left[TP\right]_h}.\left[TP\right]_h.\left[P_{mh}[if]_h+P_{mh}[rc]_h\right].\left[1-[ex]_h-[if]_h\right]-\left[\xi_h+[BIR]_m\right][ex]_m$$

$$\frac{d}{dt}\left[if\right]_m=\xi_m\left[ex\right]_m-[BIR]_m\left[if\right]_m$$

$$\frac{d}{dt}\left[TP\right]_m=[BIR]_m\left[TP\right]_m-\left([DID]_m+[DDD]_m\left[TP\right]_m\right)\left[TP\right]_m \tag{B}$$

Now, R_0 can be defined as, $R_0=\sqrt{\Re_{hm}.\Re_{mh}}$, [2],[3],[4] (C)

where $\Re_{hm}$ and $\Re_{mh}$ can be written in mathematical notation as,

$$\Re_{hm}=\frac{\xi_m}{\xi_m+[DID]_m+[DDD]_m\left[TP\right]_m^*}\,\frac{\phi_h\phi_m P_{mh}\left[TP\right]_h^*}{\phi_m\left[TP\right]_m^*+\phi_h\left[TP\right]_h^*}\,P_{mh}\left[[DID]_m+[DDD]_m\left[TP\right]_m^*\right]^{-1}$$

$$\begin{aligned}\Re_{mh}=&\frac{\xi_h}{\xi_h+[DID]_h+[DDD]_h\left[TP\right]_h^*}\,\frac{\phi_h\phi_m P_{mh}\left[TP\right]_m^*}{\phi_h\left[TP\right]_h^*+\phi_m\left[TP\right]_m^*}\left(\theta_h+\eta_h+[DID]_h+[DDD]_h\left[TP\right]_h^*\right)^{-1}\\ &.\left[P_{mh}+\overline{P_{mh}}.\theta_h\left(\eta_h+[DID]_m+[DDD]_m\left[TP\right]_h^*\right)^{-1}\right]\end{aligned} \tag{D}$$

In [2,3,4, S. Dhevarajan et.al, 2013], it was proved that the above said model is asymptotically stable. It is also proved that the domain, existence and uniqueness of the solution of the above said model. The disease free equilibrium of the model is also proved [3]. The solution of the above said model is asymptotically stable [2]. The disease free equilibrium solution is also exists [5]. Asymptotically equilibrium points are near to equilibrium points that are near to them move toward the equilibrium solution [1].

The Existence of Endemic Equilibrium Points of SPR-SODE Model

The equilibrium equations for (B) are shown below in (E). In this analysis, Here, the terms

$[ex]_h, [if]_h, [rc]_h, [TP]_h, [ex]_m, [if]_m$ and $[TP]_m$ are representing their respective equilibrium values and not their actual values at a given time t.

$$0=\frac{\phi_h\phi_m P_{mh}[if]_m}{\phi_m[TP]_m+\phi_h[TP]_h}.[TP]_m.\left[1-[ex]_h-[if]_h-[re]_h\right]-\left[\xi_h+[BIR]_h+\frac{\wp_h}{[TP]_h}\right][ex]_h+\eta_h[if]_h[ex]_h \tag{E1}$$

$$0=\xi_h[ex]_h-\left(\theta_h+[BIR]_h+\frac{\wp_h}{[TP]_h}\right)[if]_h+\eta_h[if]_h^2 \tag{E2}$$

$$0=\theta_h[if]_h-\left(L_h+[BIR]_h\frac{\wp_h}{[TP]_h}\right)[rc]_h+\eta_h[if]_h[TP]_h \tag{E3}$$

$$0=\wp_h+\theta_h[TP]_h-\left([DID]_h+[DDD]_h[TP]_h\right).[TP]_h-\eta_h[if]_h[TP]_h \tag{E4}$$

$$0=\frac{\phi_h\phi_m}{\phi_m[TP]_m+\phi_h[TP]_h}.[TP]_h.\left[P_{mh}[if]_h+P_{mh}[rc]_h\right].\left[1-[ex]_h-[if]_h\right]-\left[\xi_h+[BIR]_m\right][ex]_m \tag{E5}$$

$$0=\xi_m[ex]_m-[BIR]_m[if]_m \tag{E6}$$

$$0=[BIR]_m[TP]_m-\left([DID]_m+[DDD]_m[TP]_m\right)[TP]_m \tag{E7}$$

Define new parameter, $\Diamond=\phi_h/\phi_m$, to obtain

$$\Gamma\left[\frac{[TP]_m^*\,\theta[TP]_h^*}{[TP]_m\,\theta[TP]_h}\right][TP]_m P_{mh}.[if]_m\left[1-[ex]_h-[if]_h-[re]_h\right]-\left[\xi_h+[BIR]_h+\frac{\wp_h}{[TP]_h}\right][ex]_h+\eta_h[if]_h[ex]_h=0 \tag{E8}$$

$$\Gamma\left[\frac{[TP]_m^*\,\theta[TP]_h^*}{[TP]_m\,\theta[TP]_h}\right].[TP]_h.\left[P_{mh}[if]_h+P_{mh}[rc]_h\right].\left[1-[ex]_h-[if]_h\right]-\left[\xi_h+[BIR]_m\right][ex]_m=0 \tag{E9}$$

The bifurcation parameter Γ can be varied, while keeping all other parameters fixed. In terms of the original variables, this corresponds to changing ϕ_h and ϕ_m, while keeping the ratio between them fixed. Consider $\Diamond=\phi_h/\phi_m$. One can choose the ratio $\Diamond$ to sweep out the entire parameter space. Hence, $[TP]_m=\frac{[BIR]_m-[DID]_m}{[DDD]_m}$ (E10)

Solving for $[if]_m$ in (E6) in terms of $[ex]_m$, one can find $[if]_m=\frac{\xi_m[ex]_m}{[BIR]_m}$ (E11)

By rewriting the positive equilibrium for $[TP]_h$ in terms of $[if]_h$ from (E9) as

$$[TP]_h=\frac{\left[[BIR]_h-[DID]_h-\eta_h[if]_h\right]+\sqrt{\left[[BIR]_h-[DID]_h-\eta_h[if]_h\right]^2-4[DDD]_h\wp_h}}{2[DDD]_h} \tag{E12}$$

Using (E12) in (E3), and solving for $\left[rc\right]_h$ in terms of $\left[if\right]_h$:

$$\left[rc\right]_h = \frac{2\theta_h\left[if\right]_h}{\left(2L_h + \left[[BIR]_h + [DID]_h - \eta_h[if]_h\right]\right) + \sqrt{\left[[BIR]_h - [DID]_h - \eta_h[if]_h\right]^2 + 4[DDD]_h\wp_h}} \tag{E13}$$

It is given the nonlinear nature of (E2), which is not possible to solve for $\left[if\right]_h$ in terms of $\left[ex\right]_h$ explicitly. Now, by using (E12) rewrite (E2), and define the function $\left[ex\right]_h = g\left(\left[if\right]_h\right)$ as

$$g\left(\left[if\right]_h\right) = \frac{\theta_h + \eta_h + \frac{1}{2}\left[[[BIR]_h + [DID]_h - \eta_h[if]_h]\right] + \sqrt{\left[[BIR]_h - [DID]_h - \eta_h[if]_h\right]^2 + 4[DDD]_h\wp_h}}{\xi_h}\left[if\right]_h$$

It is clear that g (0) = 0. Coin the positive constant g (1) as $\left[ex_{\max}\right]_h$. As $g\left(\left[if\right]_h\right)$ is a nice smooth and continuous function of $\left[if\right]_h$ with $g'\left(\left[if\right]_h\right) > 0$ for $\left[if\right]_h \in [0, 1]$ and $\left[ex\right]_h \in \left[0, \left[ex_{\max}\right]_h\right]$, there exists a smooth function $\left[if\right]_h = y\left(\left[ex\right]_h\right)$ with domain $\left[0, \left[ex_{\max}\right]_h\right]$ and range [0, 1]. As g (0) > 0, the smooth function $y\left(\left[ex\right]_h\right)$ would extend to some small $\left[ex\right]_h < 0$. $[TP]_h$ and $\left[rc\right]_h$ can also be expressed as functions of $\left[ex\right]_h$ by substituting $\left[if\right]_h = y\left(\left[ex\right]_h\right)$ into (E12) and (E13). Now introduce D_1, the bounded open subset of R^2 defined by

$$D_1 = \left\{ \begin{pmatrix}\left[ex\right]_h \\ \left[ex\right]_m\end{pmatrix} \in \Re^2 \;\middle|\; \begin{matrix} -\left[ex\right]_h < \left[ex\right]_h < \left[ex_{\max}\right]_h \\ -\left[ex\right]_m < \left[ex\right]_m < 1 \end{matrix} \right\} \tag{E14}$$

for some $\left[ex\right]_h > 0$ and some $\left[ex\right]_m > 0$. Define Z as the open and bounded set Z = $\{ \Gamma \in R | -M_Z < \Gamma < M_Z \}$. This set is defined to include the characteristic values of L, so there is minimum value that M_Z can have, but M_Z may be arbitrarily large with $\Gamma = \frac{\phi_m\phi_h}{\phi_m N_m^* + \phi_h N_h^*}$.

Determination of Lower Order Terms

Now it is to determine lower order terms. (E2) can be written as $f\left(\left[ex\right]_h, \left[if\right]_h\right) = 0$, where

$f\left(\left[ex\right]_h, \left[if\right]_h\right) =$

$$\xi_h\left[ex\right]_h - \left[\theta_h + \eta_h + \frac{1}{2}\left[[[BIR]_h + [DID]_h - \eta_h[if]_h]\right] + \sqrt{\left[[BIR]_h - [DID]_h - \eta_h[if]_h\right]^2 + 4[DDD]_h\wp_h}\right]\left[if\right]_h$$

and use implicit differentiation to write $\left[if\right]_h = y\left(\left[ex\right]_h\right)$ as a Taylor polynomial of the form

$$\left[if\right]_h = \Im_1\left[ex\right]_h + O\left(\left[ex\right]_h\right)^2 \tag{E15}$$

Where $\Im_1 = \begin{bmatrix} \dfrac{\partial f}{\partial [ex]_h} \\ \dfrac{\partial f}{\partial [if]_h} \end{bmatrix}_{[ex]_h = [if]_h = 0} = \dfrac{\xi_h}{\left[\theta_h + \eta_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h} \right]}$

The first order approximations to the equilibrium equations can be obtained by substituting (E15) into (E13) and (E12), and then all three, along with (E11) and (E10) into the equilibrium equations (E8) and (E9). Hence,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f_{1\_10} & f_{1\_01} \\ f_{2\_10} & f_{2\_01} \end{pmatrix} \begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} + O\begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix}^2 \qquad \text{(E16)}$$

Where, $f_{1\_10} = -\left[\xi_h + \eta_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h} \right]$ (E17a)

$$f_{1\_01} = \Gamma \frac{\xi_m P_{hm} \cdot \left[[BIR]_m - [DID]_m\right]}{[BIR]_m [DDD]_m} \qquad \text{(E17b)}$$

$$f_{2\_10} = \Gamma . \frac{\xi_h \left[\left[[BIR]_h - [DID]_h\right] + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h} \right]}{2[DDD]_h \left(\theta_h + \eta_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] \right) + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h}} \left[P_{mh} + \frac{\theta_h \, . \overline{P}_{mh}}{\left(L_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] \right) + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h}} \right] \qquad \text{(E17c)}$$

$$f_{2\_10} = -\left([BIR]_m + \xi_m\right) \qquad \text{(E17d)}$$

To apply Corollary 1.12 of Rabinowitz [6], we algebraically manipulate (E16) to produce

$$u = \Gamma L u + h(\Gamma, u) \qquad \text{(E18)}$$

Where $u = \begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix}$ and $L = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$ With (E19a)

$$B = \frac{\xi_h \left[\left[[BIR]_h - [DID]_h\right] + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h} \right]}{2[DDD]_h \left([BIR]_m + \xi_h\right) \left[\left(\theta_h + \eta_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] \right) + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h} \right]} \left[P_{mh} + \frac{\theta_h \, . \overline{P}_{mh}}{\left(L_h + \frac{1}{2}\left[[BIR]_h + [DID]_h\right] \right) + \sqrt{\left[[BIR]_h - [DID]_h\right]^2 + 4[DDD]_h \wp_h}} \right] \qquad \text{(E.19b)}$$

and h(Γ, u) is O(u^2). The two characteristic values of L by $\lambda_1 = 1/\sqrt{AB}$ and $\lambda_2 = -1/\sqrt{AB}$. As both A and B are always positive, due to our assumption that $[BIR]_m > [DID]_m$, λ_1 is real and corresponds to the dominant eigen value of L. The right and left eigen vectors corresponding to λ_1 are respectively,

$$v = \left(\sqrt{A} \quad \sqrt{B}\right)^T \text{ and } w = \left(\sqrt{B} \quad \sqrt{A}\right) \tag{E20}$$

For $M_Z > \lambda_1$, as $0 \in Y$, $(\lambda_1, 0) \in \hat{\lambda}$. We denote the continuum of solution-pairs emanating from (λ_1, 0) by $\prod_1$, where $\prod_1 \in \hat{\lambda}$, and from ($\lambda_2$, 0) by $\prod_2$, where $\prod_2 \in \hat{\lambda}$. Let Z_1, Z_2, U_1 and U_2 are the sets defined by,

$$Z_1 = \left\{\Gamma \in Z \middle| u \text{ such that } (\Gamma, u) \in \prod_1\right\} \tag{E21a}$$

$$U_1 = \left\{u \in Y \middle| \Gamma \text{ such that } (\Gamma, u) \in \prod_1\right\} \tag{E21b}$$

$$Z_2 = \left\{\Gamma \in Z \middle| u \text{ such that } (\Gamma, u) \in \prod_2\right\} \tag{E21c}$$

$$U_2 = \left\{u \in Y \middle| \Gamma \text{ such that } (\Gamma, u) \in \prod_2\right\}$$ (E21d) [Change the variable, Y and Z]

The part of Y in the positive quadrant of R^2 can be denoted by Y^+ and defined by $Y^+ = \left\{\left([ex]_h, [ex]_m\right) \in Y \;\middle|\; [ex]_h > 0 \text{ and } [ex]_m > 0\right\}$. The internal boundary of Y^+ can be defined by

$$\partial Y^+ = \left\{\begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} \in Y \;\middle|\; \begin{pmatrix} [ex]_h > 0 \text{ and} \\ [ex]_m = 0 \end{pmatrix} \text{or} \begin{pmatrix} [ex]_h = 0 \text{ and} \\ [ex]_m > 0 \end{pmatrix} \text{or} \begin{pmatrix} [ex]_h = 0 \text{ and} \\ [ex]_m = 0 \end{pmatrix}\right\}$$

Now, it is to expand the terms of the nonlinear eigen value equation (E18) about the bifurcation point, (λ_1, 0). The expanded variables are

$$u = 0 + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots \quad \ldots\ldots \text{(E22a)}; \quad \Gamma = \lambda_1 + \varepsilon\Gamma_1 + \varepsilon^2\Gamma_2 + \ldots \tag{E22b}$$

$$\begin{aligned} h(\Gamma, u) &= h\left(\lambda_1 + \varepsilon\Gamma_1 + \varepsilon^2\Gamma_2 + \ldots, \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots\right) \\ &= \varepsilon^2 h_2\left(\lambda_1, u^{(1)}\right) + \ldots. \end{aligned} \tag{E22c}$$

Now, consider, $\Gamma_1 = -\dfrac{w.h_2}{w.Lv}$ (E23)

Now it is to prove that if v and w are the right and left eigenvectors of L corresponding to the characteristic value λ_1, respectively, the bifurcation at $\Gamma = \lambda_1$ of the nonlinear eigen value equation (E18) is supercritical if $\Gamma_1 > 0$ and subcritical if $\Gamma_1 < 0$.

Evaluating the substitution of the expansions (E22) into the eigen value equation (E18) at $O(\varepsilon^2)$, we obtain $u^{(2)} = \lambda_1 Lu^{(2)} + \Gamma_1 Lu^{(1)} + h_2$, which we can rewrite as

$$(I - \lambda_1 L)\, u^{(2)} = \Gamma_1 Lv + h_2, \tag{E24}$$

Where I is the 2 × 2 identity matrix. As λ_1 is a characteristic value of L, $(I - \lambda_1 L)$ is a singular matrix. Thus, for (E24) to have a solution, $\Gamma_1 Lv + h_2$ must be in the range of $(I - \lambda_1 L)$; i.e., it must be orthogonal to the null space of the

adjoint of (I− λ_1 L). The null space of the adjoint of (I− λ_1 L) is spanned by the left eigenvector of L (corresponding to the eigen value 1/ λ_1), w (E20). The Fredholm condition for the solvability of (A.24) is w. (Γ_1 Lv + h_2) = 0. Solving for Γ_1 provides (E23) obviously. If Γ_1 is positive, then for small positive ε, u > 0 and $\Gamma > \lambda_1$, and the bifurcation is supercritical. Similarly, if Γ_1 is negative, then for small positive ε, u > 0 and $\Gamma < \lambda_1$, and the bifurcation is subcritical.

Now, it is to prove that For all u $\in U_1$, $[ex]_h > 0$ and $[ex]_m > 0$.

From [5], there are no equilibrium points on ∂Y^+ other than $[ex]_h = [ex]_m = 0$, so U1 ∩ ∂Y^+ = 0. From [5], close to the bifurcation point (λ_1, 0), the direction of U_1 is equal to v, the right eigenvector corresponding to the characteristic value, λ_1. As v contains only positive terms, U_1 is entirely contained in Y^+. Hence, for all u $\in U_1$, $[ex]_h > 0$ and $[ex]_m > 0$.

Equilibrium in the Boundary of the Positive Orthant

Now it is to prove that the point u = 0 ∈ Y corresponds to $x_{nodis} \in R^7$ (on the boundary of the positive orthant of R^7). For every solution-pair (Γ, u) $\in \prod_1$, there corresponds one equilibrium-pair (Γ, x^*) $\in Z \times R^7$, where x^* is in the positive orthant of R^7.

First it is to show that u = 0 corresponds to x_{nodis}. As $[ex]_h = [ex]_m = 0$, From [1] & [6], One can argue that the only possible equilibrium point is x_{nodis}. Now it is to show that for every $\Gamma \in Z_1$ there exists an x^* in the positive orthant of R^7 for the corresponding u $\in U_1$. (E25) implies $[ex]_h > 0$ and $[ex]_m > 0$. The equilibrium equation (E11) implies for every positive $[ex]_m$, there exists a positive $[if]_m$. The equilibrium equation for $[TP]_m$ has a positive and bounded solution, depending only on parameter values (E10). Now, $[if]_h = y\left([ex]_h\right)$ implies for every positive $[ex]_h$ there exists a positive $[if]_h$. The equilibrium equations (E13) and (E12) show that, for every positive $[if]_h$ there exists a positive $[rc]_h$ and $[TP]_h$. Hence, the point u = 0 ∈ Y corresponds to $x_{nodis} \in R^7$ (on the boundary of the positive orthant of R^7). For every solution-pair (Γ, u) $\in \prod_1$, there corresponds one equilibrium-pair (Γ, x^*) $\in Z \times R^7$, where x^* is in the positive orthant of R^7…….(E26)

CONCLUSIONS

The different parameters of the SPR_SODE model are considered for further analysis. Here it is proved that the bifurcation at the characteristic value of the non-linear eigen value equation is supercritical if $\Gamma_1 > 0$ and subcritical if $\Gamma_1 < 0$. $[TP]_h$. It is also proved that the point u = 0 ∈ Y corresponds to $x_{nodis} \in R^7$, on the boundary of the positive orthant of R^7) and For every solution-pair (Γ, u) $\in \prod_1$, there corresponds one equilibrium-pair (Γ, x^*) $\in Z \times R^7$, where x^* is in the positive orthant of R^7.

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